Calculus 1 (BE5B01MA1)
Basic rules: derivation & integration

Derivation

(1) $(u + kv)' = u' + kv'$, k constant

(2) $(u \cdot v)' = u'v + uv'$

(3) $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$

(4) $(x^p)' = px^{p-1}$

(5) $[f(g(x))]' = f'(g(x))g'(x)$

(6) $\frac{df}{dx} = \frac{df}{du} \cdot \frac{du}{dx}$

Derivatives os some basic functions

(1) $(\ln(x))' = \frac{1}{x}$

(2) $(e^x)' = e^x$

(3) $(\sin(x))' = \cos(x)$.

(4) $(\cos(x))' = -\sin(x)$.

(5) $(\tan(x))' = \sec^2(x)$

(6) $(\sec(x))' = \sec(x)\tan(x)$

(7) $(\cot(x))' = -\csc^2(x)$

(8) $(\csc(x))' = -\csc(x)\cot(x)$.

Integration

(1) $\int cf(x)dx = c\int f(x)dx$

(2) $\int kdx = kx + C$

(3) $\int x^n dx = \frac{x^{n+1}}{n+1} + C \ (n \neq -1)$

(4) $\int \sin(x)dx = -\cos(x) + C$

(5) $\int \cos(x)dx = \sin(x) + C$

(6) $\int a^x dx = \frac{a^x}{\ln(a)} + C$ and $a \neq 1$

(7) $\int \frac{1}{x^2 + a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

(8) $\int \sec^2(x)dx = \tan(x) + C$

(9) $\int \csc^2(x)dx = -\cot(x) + C$

(10) $\int \sec(x)\tan(x)dx = \sec(x) + C$

(11) $\int \csc(x)\cot(x)dx = -\csc(x) + C$

(12) $\int e^x dx = e^x + C$

(13) $\int \frac{1}{x}dx = \ln(|x|) + C$

(14) $\int \sec(x)dx = \ln(|\tan(x) + \sec(x)|) + C$

Trigonometric identities

(1) $\sin^2(x) + \cos^2(x) = 1$

(2) $\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$

(3) $\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$

(4) $\sin(x)\cos(x) = \frac{1}{2}\sin(2x)$

(5) $\sec^2(x) = 1 + \tan^2(x)$

(6) $1 + \cot^2(x) = \csc^2(x)$

(7) $\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$

(8) $\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$

(9) $\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$

(10) $\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$

(11) $\tan(x+y) = \dfrac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$

(12) $\tan(x-y) = \dfrac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$

Even/Odd Formulas

(1) $\sin(-x) = -\sin(x)$

(2) $\cos(-x) = \cos(x)$

(3) $\tan(-x) = -\tan(x)$

(4) $\csc(-x) = -\csc(x)$

(5) $\sec(-x) = \sec(x)$

(6) $\cot(-x) = -\cot(x)$

Table of Trigonometric Substitutions

Expression	Substitution	Identity
$\sqrt{a^2 - x^2}$	$x = a\sin\theta, \quad -\frac{\pi}{2} \leqslant \theta \leqslant \frac{\pi}{2}$	$1 - \sin^2\theta = \cos^2\theta$
$\sqrt{a^2 + x^2}$	$x = a\tan\theta, \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$	$1 + \tan^2\theta = \sec^2\theta$
$\sqrt{x^2 - a^2}$	$x = a\sec\theta, \quad 0 \leqslant \theta < \frac{\pi}{2}$ or $\pi \leqslant \theta < \frac{3\pi}{2}$	$\sec^2\theta - 1 = \tan^2\theta$

Some Maclaurin series

- ⋆ $\dfrac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \ldots$ with $R = 1$.

- ⋆ $e^x = \sum_{n=0}^{\infty} \dfrac{x^n}{n!} = 1 + \dfrac{x}{1!} + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \ldots$ with $R = \infty$.

- ⋆ $\sin(x) = \sum_{n=0}^{\infty} (-1)^n \dfrac{x^{2n+1}}{(2n+1)!} = x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \dfrac{x^7}{7!} + \ldots$ with $R = \infty$.

- ⋆ $\cos(x) = \sum_{n=0}^{\infty} (-1)^n \dfrac{x^{2n}}{(2n)!} = 1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!} - \dfrac{x^6}{6!} + \ldots$ with $R = \infty$.

- ⋆ $\tan^{-1}(x) = \sum_{n=0}^{\infty} (-1)^n \dfrac{x^{2n+1}}{2n+1} = x - \dfrac{x^3}{3} + \dfrac{x^5}{5} - \dfrac{x^7}{7} + \ldots$ with $R = 1$.

- ⋆ $\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \dfrac{x^n}{n} = x - \dfrac{x^2}{2} + \dfrac{x^3}{3} - \dfrac{x^4}{4} + \ldots$ with $R = 1$.

- ⋆ $(1+x)^k = 1 + kx + \dfrac{k(k-1)}{2!} x^2 + \dfrac{k(k-1)(k-2)}{3!} x^2 + \ldots$ with $R = 1$.