CALCULUS 1 (BE5B01MA1)
LAB 11 & 12

Exercise 1. Give an example of a sequence $(a_n)_{n=1}^{\infty}$ not constant such that it is:

(1) bounded and increasing.
(2) bounded and decreasing.
(3) bounded and non-monotonic.
(4) unbounded and non-increasing.
(5) unbounded and non-monotonic.
(6) monotonic and unbounded.

Exercise 2. Decide whether the sequence converges or diverges. If it converges, evaluate the limit.

(1) $a_n = \dfrac{n-1}{n+1}$
(2) $a_n = \dfrac{\ln(n)}{e^n}$
(3) $a_n = \dfrac{4n^2 - 3n}{n^2 + 5n - 6}$
(4) $a_n = \dfrac{n^2}{n+1} - \dfrac{n^2}{n+2}$
(5) $a_n = \dfrac{n}{e^n}$
(6) $a_n = \dfrac{3n\sqrt{n}+1}{7-2n\sqrt{n}}$
(7) $a_n = \tan\left(\dfrac{2n\pi}{1+8n}\right)$
(8) $a_n = \dfrac{(-1)^n}{2\sqrt{n}}$
(9) $a_n = \cos\left(\dfrac{2}{n}\right)$
(10) $a_n = \ln(n+1) - \ln(n)$.
(11) $a_n = \{0, 1, 0, 1, 0, 1, \dots\}$
(12) $a_n = \dfrac{\sin(2n)}{1+\sqrt{n}}$

Exercise 3 (Ratio Test)**.** Show that the following sequences converge to zero.

$$a_n = \frac{n!}{n^n}, \quad b_n = \frac{r^n}{n!}, \quad c_n = \frac{(-1)^n n!}{1 \cdot 3 \cdot 5 \cdot \ldots \cdot (2n-1)}, \quad d_n = \frac{n^p}{2^n}.$$

Exercise 4. Show that

$$\lim_{n\to\infty} (3^n + 4^n)^{\frac{1}{n}} = 4.$$

Use the fact that $\lim_{n\to\infty} \left(\dfrac{3}{4}\right)^n = 0$.

Exercise 5. More sequences to decide if they are convergent or divergent.

(1) $a_n = \dfrac{2^n}{n!}$

(2) $a_n = \dfrac{n^n}{n!}$

(3) $a_n = \dfrac{2^n}{1+2^n}$

(4) $a_n = \dfrac{1 \cdot 3 \cdot 5 \cdot \ldots \cdot (2n-1)}{n! 2^n}$

(5) $a_n = \dfrac{n^2}{\ln(n+1)}$

(6) $a_n = \dfrac{n}{2^n} + \dfrac{(-1)^n}{n}$

(7) $a_n = \ln(e^n - 1) - n$

(8) $a_n = \dfrac{n^2}{2n-1} - \dfrac{n^2}{2n+1}$

Exercise 6 (Group discussion)**.** True **(T)** or false **(F)**? If it is (T), explain why. If it is (F), give a counterexample.

(   ) Every bounded sequence is convergent.

(   ) Every bounded sequence is monotonic.

(   ) Every monotonic sequence is convergent.

(   ) Every bounded decreasing sequence is convergent and its limit is zero.

(   ) If $(a_n)_{n=1}^{\infty}$ is a divergent sequence, then so is $(|a_n|)_{n=1}^{\infty}$.

(   ) If $(a_n)_{n=1}^{\infty}$ is a convergent sequence, then so is $((-1)^n a_n)_{n=1}^{\infty}$.

(   ) If $(|a_n|)_{n=1}^{\infty}$ converges to zero, then $(a_n)_{n=1}^{\infty}$ also converges to zero.