Calculus 1 (BE5B01MA1)
Lab 13

Exercise 1 (Power series). Find the radius of convergence and interval of convergence of the following series.

(1) $\displaystyle\sum_{n=1}^{\infty} n^n (x-3)^n$

(2) $\displaystyle\sum_{n=1}^{\infty} \frac{x^{2n+1}}{(-4)^n}$

(3) $\displaystyle\sum_{n=0}^{\infty} \frac{1\cdot 3\cdot 5\cdot \ldots \cdot (2n-1)\cdot x^{2n+1}}{2\cdot 4\cdot \ldots \cdot (2n)}$

(4) $\displaystyle\sum_{n=0}^{\infty} (-1)^{n+1} x^n$

(5) $\displaystyle\sum_{n=0}^{\infty} \frac{n(x-1)^{2n}}{3^{2n-1}}$

(6) $\displaystyle\sum_{n=1}^{\infty} \frac{1\cdot 3\cdot 5\cdot \ldots \cdot (2n-1)\cdot x^{n}}{2\cdot 4\cdot 6\cdot \ldots \cdot (2n)}$

(7) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n x^n}{n\ln(n)}$

(8) $\displaystyle\sum_{n=1}^{\infty} \frac{(x+5)^{n-1}}{n^2}$

(9) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^{2n-1}}{(2n-1)!}$

(10) $\displaystyle\sum_{n=0}^{\infty} n! x^n$

Exercise 2 (Power series). If $k \in \mathbb{N}$, find the radius of convergence of the series $\displaystyle\sum_{n=0}^{\infty} \frac{(n!)^k}{(kn)!} x^n$.

Exercise 3 (Power series). Let $f(x) = \displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n^2}$. Find the intervals of convergence for f, f', f''.

Exercise 4. (a). Use differentiation to find a power series representation for

$$f(x) = \frac{1}{(1+x)^2}.$$

What is the radius of convergence?

(b). Use part (a) to find a power series for

$$f(x) = \frac{1}{(1+x)^3}.$$

(c). Use part (b) to find a power series for

$$\frac{x^2}{(1+x)^3}.$$

Exercise 5. Find a power series representation for the function and determine the radius of convergence.

(1) $f(x) = \ln(5 - x)$

(2) $f(x) = x^2 \tan^{-1}(x^3)$

(3) $f(x) = \dfrac{x}{(1+4x)^2}$

(4) $f(x) = \left(\dfrac{x}{2-x}\right)^3$

(5) $\dfrac{1+x}{(1-x)^2}$

(6) $\dfrac{x^2+x}{(1-x)^3}$

Exercise 6. Evaluate the indefinite integrals as a power series. Find the radius of convergence.

$$(1)\ \int \frac{t}{1-t^8}dt \quad (2)\ \int \frac{t}{1+t^3}dt \quad (3)\ \int x^2 \ln(1+x)dx \quad (4)\ \int \frac{\tan^{-1}(x)}{x}dx.$$

Exercise 7. Consider the Bessel function

$$J_0(x) = \sum_{n=1}^{\infty} \frac{(-1)^n x^{2n}}{2^{2n}(n!)^2},$$

which is defined for all x. Show that J_0 satisfies

$$x^2 J_0''(x) + xJ_0'(x) + x^2 J_0(x) = 0.$$

Exercise 8 (Group discussion)**.**

(a) Find a power series representation for $f(x) = \ln(1-x)$.

(b) What is the radius of convergence for item (a)?

(c) Use part (a) to find a power series for $f(x) = x\ln(1-x)$.

(d) Use (a) to express $\ln(2)$ as the sum of an infinite series.