Calculus 1 (BE5B01MA1)
Lab 3

**Exercise 1.** In each case, find the derivative of the function $y = f(x)$ by using the definition.

(a) $y = x^2 + 1$.
(b) $y = 2x^3$.
(c) $y = x^2 - 5$.
(d) $y = 2x^2 - 3x$.
(e) $y = \dfrac{1}{x+1}$.

**Exercise 2.** Calculate the derivative of the following functions with the techniques we have learned so far.

(a) $f(x) = \dfrac{\pi}{x} + \ln(2)$. $\left(f'(x) = -\dfrac{\pi}{x^2}\right)$
(b) $g(x) = 2x + 5\cos^3(x)$. $\left(g'(x) = 2 - 15\cos^2(x)\sin(x)\right)$
(c) $h(x) = (3 - 2\sin(x))^5$. $\left(h'(x) = -10(3 - 2\sin(x))^4\cos(x)\right)$
(d) $k(x) = \sqrt{\dfrac{3\sin(x) - 2\cos(x)}{5}}$. $\left(k'(x) = \dfrac{3\cos(x) + 2\sin(x)}{2\sqrt{15\sin(x) - 10\cos(x)}}\right)$

**Exercise 3.** If $h(x) = xg(x)$ and it is known that $g(3) = 5$ and $g'(3) = 2$, find $h'(3)$.
(Apply the product rule)

**Exercise 4.** Determine the equation of the normal line to the curve $y = -x^3/6$ with slope $m = 8/9$.
(The normal line is perpendicular to the tangent line: its slope is the negative reciprocal of $m$)

**Exercise 5.** Consider $f$ and $g$ defined as follows.

$$f(x) = \begin{cases} x^2, & \text{if } x \leqslant 1, \\ 2, & \text{if } x > 1. \end{cases} \quad \text{and} \quad g(x) = \begin{cases} x^2, & \text{if } x \leqslant 1, \\ 1, & \text{if } x > 1. \end{cases}$$

(a) Draw the graphs of $f$ and $g$.
(b) $f$ and $g$ are continuous at $x = 1$?
(c) $f$ and $g$ are differentiable at $x = 1$?

**Exercise 6.** Suppose that $f$ is differentiable in $\mathbb{R}$ and it satisfies

$$f(a+b) = f(a) + f(b) + 5ab$$

for every $a, b \in \mathbb{R}$. If

$$\lim_{h \to 0} \frac{f(h)}{h} = 3,$$

then calculate $f(0)$ and $f'(x)$.
(To calculate $f(0)$, notice that $f(0) = f(0+0)$. To calculate $f'(x)$, apply the definition)

**Exercise 7.** Let $f$ be a function defined on $\mathbb{R}$ and given by $f(x) = x|x|$ for every $x \in \mathbb{R}$.

(a) Calculate $f'(x)$ when $x \neq 0$.
(b) Decide if $f'(0)$ exists.

**Exercise 8.** The position of a particle is given by the equation

$$s(t) = \frac{1}{3}t^3 - t^2 - 3t.$$

(a) Find the expressions for the velocity and acceleration of the particle.
(b) In what moment the velocity is zero?
(c) In what moment the acceleration is zero?

((a). $v(t) = t^2 - 2t - 3$ and $a(t) = 2t - 2$; (b). $t = 3$; (c). $t = 1$))

**Exercise 9** (group discussion)**.** Let $f$ be a differentiable function. If

$$h(x) = [f(x)]^3 + f(x^3),$$

calculate $h'(2)$ by knowing that $f(2) = 1$, $f'(2) = 7$, and $f'(8) = -3$.