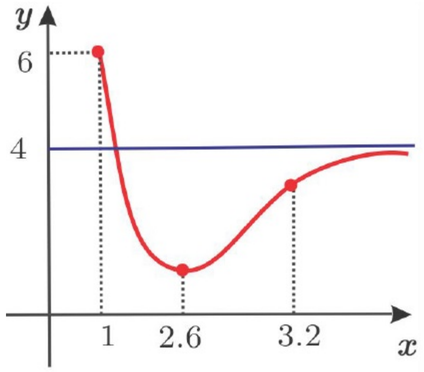
CALCULUS 1 (BE5B01MA1)
LAB 4

Exercise 1. Prove that the equation $x^3 + x - 1 = 0$ has **exactly** on real root.

Exercise 2. Prove that there exists only one real number x such that $e^x + x = 0$.

Exercise 3. Let $f : [a, b] \to \mathbb{R}$ be a continuous function. If f is differentiable on (a, b) and $f'(x) = 0$ for every $x \in (a, b)$, show that f is constant in (a, b).

Exercise 4. Consider the following graph of a function f, which is differentiable on $[1, 4)$, and answer the questions.



(a) In what interval f is increasing?

(b) In what interval f is decreasing?

(c) f has a maximum or a minimum? How about an inflection point?

(d) Calculate $\lim_{x \to \infty} f(x)$.

Exercise 5. Consider the function $f(x) = 2x^3 - 9x^2 + 12x - 3$.

(a) Decide the existence of local maximum and minimum of f.

(b) Calculate the maximum and minimum values of f in the interval $[0, 3]$.

(c) Repeat item (b) for the interval $[1/2, 3]$.

(d) Repeat item (b) for the interval $[1/2, 5/2]$.

(e) Repeat item (b) for the interval $[3/4, 9/4]$.

Exercise 6. Sketch the graph of the following functions.

(a) $f(x) = x^3 - 3x$.

(b) $g(x) = x^4 - 2x^2$.

Exercise 7 (Group discussion)**.** Let a, b, c real numbers. Suppose that $a^2 < 3b$ and consider the function

$$f(x) = x^3 + ax^2 + bx + c.$$

(a) Prove that f does not have critical points.

(b) Conclude that f does not have neither maximum non minimum.

Keys and hints

Exercise 1: Use the Intermediate Value Theorem and then the Rolle's Theorem.

Exercise 2: Use the Intermediate Value Theorem to prove the existence and then the Rolle's Theorem to prove the uniqueness.

Exercise 3: Use the Mean Value Theorem