Calculus 1 (BE5B01MA1)
Labs 6 & 7

Exercise 1. Solve the following problems.

(a) Find all functions g such that $g'(x) = 4\sin(x) + \dfrac{2x^5 - \sqrt{x}}{x}$.

(b) Find f if $f'(x) = x\sqrt{x}$ and $f(1) = 2$.

(c) Find f if $f''(x) = 12x^2 + 6x - 4$, $f(0) = 4$, and $f(1) = 1$.

Exercise 2. Suppose that a differentiable function f satisfies $f(x) > 0$ for every x and $f(1) = 1$. If $f'(x) = xf(x)$, find $f(x)$.

(Derive the function $g(x) = \ln(f(x))$).

Exercise 3. Evaluate the indefinite integrals $\int f(x)dx$.

(a) $f(x) = x^3 - 5x$.

(b) $f(x) = \dfrac{1}{x^3} + \dfrac{1}{x^2} + \dfrac{1}{x}$.

(c) $f(x) = 2\sin(x) - \dfrac{1}{x^5}$.

(d) $f(x) = (1 + x^2)^2$

(e) $f(x) = \sqrt{x} + \sec^2(x)..$

(f) $f(x) = x^3\sqrt{x}$.

(g) $f(x) = f(x) = 2 + \sin^2(x)$.

(h) $f(x) = f(x) = (x + 1)x^{-1}$.

Exercise 4. Determine the function f that satisfies:

(i) $f''(x) = x^2 + e^x$,

(ii) $f(0) = 2$.

(iii) $f'(0) = 1$.

Exercise 5. Suppose that $f : [-a, a] \to \mathbb{R}$ is an even function and that $g : [-a, a] \to \mathbb{R}$ is an odd function. Show that

(a) $\displaystyle\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$.

(b) $\displaystyle\int_{-a}^{a} g(x)dx = 0$.

Exercise 6. Let f, g be differentiable functions on $\mathbb{R}$ and suppose that $f(0) = 0$ and $g(0) = 1$. If $f'(x) = g(x)$ and $g'(x) = -f(x)$ for every x, show that the function

$$h(x) = [f(x) - \sin(x)]^2 + [g(x) - \cos(x)]^2$$

has derivative zero and then it is constant. From that, conclude that

$$f(x) = \sin(x) \quad \text{and} \quad g(x) = \cos(x).$$

Exercise 7. Evaluate the definite integral of the function f on the interval I.

(a) $I = [-2, 2]$ and $f(x) = |x - 1|$.

(b) $I = [-\pi, \pi]$ and $f(x) = x + |\cos(x)|$.

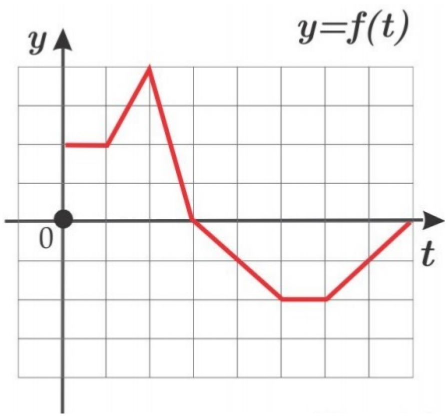
(c) $I = [-\pi, \pi]$ and $f(x) = x - |x|$.

(d) $I = [-3, 5]$ and $f(x) = |x^2 - 3x + 2|$.

(e) $I = [-\pi, \pi]$ and $f(x) = |\sin(x)|$.

(f) $I = [-1, 1]$ and $f(x) = x$ if $x < 0$ and $f(x) = x^2 - x + 1$ if $x \geqslant 0$.

Exercise 8. Consider a function $y = f(x)$ with the following graph.



Define the function g by

$$g(x) = \int_0^x f(t)dt.$$

(a) Evaluate $g(0)$, $g(1)$, $g(2)$, $g(3)$, and $g(6)$.

(b) Determine the interval that g is increasing.

(c) When g attains its maximum?

Exercise 9 (Group discussion)**.** Consider $f : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = \begin{cases} 2 + x^3/4 & , \text{ if } x < 0, \\ x^2 - x - 2 \,, & \text{if } 0 \leqslant x < 3, \\ 16 - 4x & , \text{ if } x \geqslant 3. \end{cases}$$

(a) Sketch the graph of f.

(b) Find the area A that lies under $y = f(x)$, the x-axis, $x = -2$ and $x = 5$.

(c) Evaluate $I = \displaystyle\int_{-2}^{5} f(x)dx$.

(d) Why is $I \neq A$?