CALCULUS 1 (BE5B01MA1)
LAB 7

Exercise 1. Evaluate the integral.

(1) $\int_{-2}^{3}(x^2-3)dx.$ $\left(=\frac{-10}{3}\right)$

(2) $\int_{-2}^{0}\left(\frac{t^4}{2}+\frac{t^3}{4}-t\right)dt.$ $\left(=\frac{21}{5}\right)$

(3) $\int_{0}^{2}(2x-3)(4x^2+1)dx.$ $(=-2)$

(4) $\int_{0}^{\pi}(4\sin(\theta)-3\cos(\theta))d\theta.$ $(=8)$

(5) $\int_{-1}^{1}e^{-x}dx$ $\left(=\frac{(e-1)(e+1)}{e}\right).$

(6) $\int_{1}^{4}\left(\frac{4+6u}{\sqrt{u}}\right)du.$ $(=36)$

(7) $\int_{0}^{1}x(\sqrt[3]{x}+\sqrt[4]{x})dx.$ $\left(=\frac{55}{63}\right)$

(8) $\int_{1}^{4}\sqrt{\frac{5}{x}}dx.$ $(=2\sqrt{5})$

(9) $\int_{0}^{\pi/3}\sin(2x)dx.$ $\left(=\frac{3}{4}\right)$

(10) $\int_{-\pi/2}^{\pi/2}\cos\left(\frac{x}{2}\right)dx.$ $(=2\sqrt{2})$

(11) $\int_{0}^{\pi/4}\sec^2(t)dt.$ $(=1)$

(12) $\int_{0}^{\pi/3}\frac{\sin(\theta)+\sin(\theta)\tan^2(\theta)}{\sec^2(\theta)}d\theta.$ $\left(=1+\frac{\pi}{4}\right)$

(13) $\int_{1}^{2}(1+2y)^2dy.$ $\left(=\frac{49}{3}\right)$

(14) $\int_{1}^{64}\frac{1+\sqrt[3]{x}}{\sqrt{x}}dx.$ $\left(=\frac{256}{5}\right)$

(15) $\int_{0}^{1}(\sqrt[4]{x^5}+\sqrt[5]{x^4})dx.$ $(=1)$

(16) $\int_{1}^{4}\sqrt{t}(1+t)dt.$ $\left(=\frac{256}{15}\right)$

(17) $\int_{0}^{1}e^{2x}dx.$ $\left(=\frac{1}{2}(e^2-1)\right)$

(18) $\int_{1}^{2}\left(x+\frac{1}{x}\right)dx.$ $\left(=\frac{3}{2}+\ln(2)\right)$

(19) $\int_{0}^{\pi/3}(\sin(3x)+\cos(3x))dx$ $\left(=\frac{2}{3}\right)$

(20) $\int_{0}^{\pi/2}\left(\frac{1}{2}+\frac{1}{2}\cos(2x)\right)dx$ $\left(=\frac{\pi}{4}\right)$

Exercise 2. Show that

$$\sin^2(x)=\frac{1}{2}-\frac{1}{2}\cos(2x).$$

Now evaluate $\int \sin^2(x)dx$.

(Recall that $\cos(a+b)=\cos(a)\cos(b)-\sin(a)\sin(b)$).

Exercise 3. Evaluate the integral and interpret it as a difference of areas. Illustrate with a sketch.

(a) $\int_{-1}^{2}x^3dx.$

(b) $\int_{\pi/4}^{5\pi/2}\sin(x).$

Exercise 4. Verify by differentiation that the formula is correct.

(a) $\displaystyle\int \frac{x}{\sqrt{x^2+1}}dx = \sqrt{x^2+1} + C.$

(b) $\displaystyle\int \cos^2(x)dx = \frac{x}{2} + \frac{\sin(2x)}{4} + C.$

Exercise 5. Find the indefinite integral.

(a) $\displaystyle\int (x^2 + x^{-2})dx.$

(b) $\displaystyle\int (\sqrt{x^3} + \sqrt[3]{x^2})dx.$

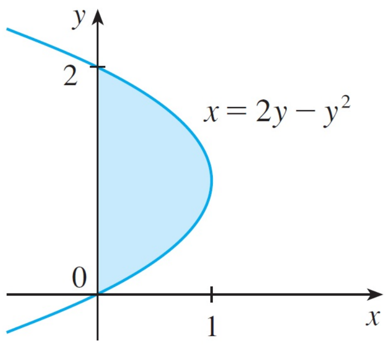
(c) $\displaystyle\int (u+4)(2u+1)du.$

(d) $\displaystyle\int v(v^2+2)^2 dv.$

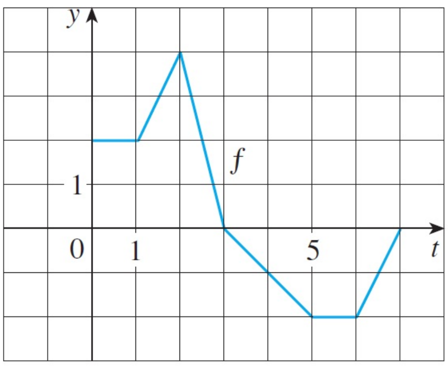
(e) $\displaystyle\int \frac{\sin(x)}{1-\sin^2(x)}dx$

(f) $\displaystyle\int \frac{\sin(2x)}{\sin(x)}dx.$

Exercise 6. Find the area of the shaded region.



Exercise 7 (Group discussion)**.** Let $g(x) = \int_0^x f(t)dt$, where f is the function whose graph is shown.



(a) Evaluate $g(0), g(1), g(2), g(3)$, and $g(6)$.
(b) On what interval is g increasing?
(c) Where does g have a maximum value?
(d) Sketch a rough graph of g.