

CALCULUS 1 (BE5B01MA1)
LAB 9

Exercise 1 (Trigonometric integrals). Evaluate the indefinite integrals.

(1) • $\int \sin^2(x) \cos^3(x) dx.$

(7) •••• $\int \tan(x) \sec^3(x) dx.$

(2) $\int \sin^3(x) \cos^4(x) dx.$

(8) $\int \tan^2(x) dx.$

(3) •• $\int_0^{\pi/2} \sin^5(x) dx.$

(9) ••••• $\int \tan^4(x) \sec^6(x) dx.$

(4) $\int_0^{\pi/2} \cos^2(x) dx.$

(10) $\int \tan^5(x) dx.$

(5) ••• $\int x \sin^2(x) dx$

(11) •••••• $\int \tan^2(x) \sec(x) dx.$

(6) $\int x \cos^2(x) dx.$

(12) $\int \csc(x) dx.$

Exercise 2. • Let $a, b \in \mathbb{R}$. Show that the area enclosed by the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

is πab .

Exercise 3 (Trigonometric substitutions). Evaluate the indefinite integrals.

(1) • $\int_0^1 x^2 \sqrt{1-x^2} dx.$

(4) $\int_0^2 x^3 \sqrt{x^2+4} dx.$

(2) $\int_{\sqrt{2}}^2 \frac{1}{t^3 \sqrt{t^2-1}} dt.$

(5) ••• $\int_0^{0.6} \frac{x^2}{\sqrt{9-25x^2}} dx.$

(3) •• $\int_0^a \frac{1}{(a^2+x^2)^{3/2}} dx, a > 0$

(6) $\int_0^1 \frac{1}{(x^2+1)^2} dx$

Exercise 4. • (a). Use the formulas for $\cos(A+B)$ and $\cos(A-B)$ to show that

$$\sin(A) \sin(B) = \frac{1}{2} [\cos(A-B) - \cos(A+B)].$$

•• (b). Use part (a) to show that

$$\int \sin(5x) \sin(2x) dx = \frac{1}{6} \sin(3x) - \frac{1}{14} \sin(7x) + C.$$

Exercise 5 (Partial fractions). Evaluate the indefinite integrals.

$$(1) \int \frac{x^4}{x-1} dx.$$

$$(2) \int \frac{5x+1}{(2x+1)(x-1)} dx.$$

$$(3) \int_0^1 \frac{2}{2x^2+3x+1} dx.$$

$$(4) \int \frac{ax}{x^2-bx} dx.$$

$$(5) \int_0^1 \frac{2x+3}{(x+1)^2} dx$$

$$(6) \int_1^2 \frac{4y^2-7y-12}{y(y+2)(y-3)} dy.$$

$$(7) \int \frac{3t-2}{t+1} dt.$$

$$(8) \int \frac{y}{(y+4)(2y-1)} dy.$$

$$(9) \int \frac{x-4}{x^2-5x+6} dx.$$

$$(10) \int \frac{1}{(x+a)(x+b)} dx.$$

$$(11) \int_0^1 \frac{x^3-4x-10}{x^2-x-6} dx.$$

$$(12) \int \frac{x^2+2x-1}{x^3-x} dx.$$

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