Calculus 1 (BE5B01MA1)
Midterm 2 (Week 11. 2019, Dec, 4th)

⋆ *This test has* **5 exercises** *with a total of* **15 points**.
⋆ *The maximum grade is* **15 points**.

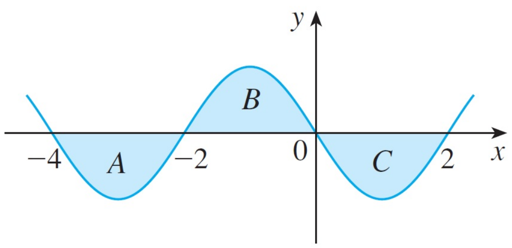
Name: ____

Exercise 1. [3pts] Consider the regions A, B, and C bounded by the graph of f and the x-axis as in the following figure.

(a) **(1pt)** If A, B, and C have each area 3, calculate the total area on the interval $[-4, 2]$.

(b) **(1pt)** Evaluate $\int_{-4}^{2} f(x)dx$.

(c) **(1pt)** Conclude that $\int_{-4}^{2} [f(x) + 2x + 5]dx = 15$.



Exercise 2. [3pts] Answer the following questions *justifying your answers.*

(a) **(1.5pts)** Show that $\int xe^x dx = xe^x - e^x + C$.

(b) **(1.5pts)** Using that $\lim_{t\to-\infty} e^t = \lim_{t\to-\infty} te^t = 0$, conclude that $\int_{-\infty}^{0} xe^x dx = -1$.

Exercise 3. [3pts] Consider the following integral $\int \sqrt{2x+1}dx$.

(a) **(1.5pts)** Let $u = 2x + 1$ and conclude that $\int \sqrt{2x+1}dx = \frac{1}{3}(2x+1)^{3/2} + C$.

(b) **(1.5pts)** Now, let $u = \sqrt{2x+1}$ and conclude the same result as in (a).

Exercise 4. [3pts] Decide if the following statements are **true** or **false**.

- If it is **true**, explain why.
- If it is **false**, give an example that disproves the statement.

(a) **(0.75pts)** Since the integral $\int_1^\infty \frac{1}{x^2}dx$ is convergent, so is $\int_1^\infty \frac{1}{x}dx$.

(b) **(0.75pts)** The definite integral $\int_{-\pi/4}^{\pi/4} \frac{x^4 \tan(x)}{2+\cos(x)}dx$ is zero.

(c) **(0.75pts)** If f is an even continuous function on $[-a, a]$, then $\int_{-a}^{a} f(x)dx = 0$.

(d) **(0.75pts)** The integral of a continuous function can *always* be interpreted as an area.

Exercise 5. [3pts]

(a) **(1.5pts)** By using partial fractions, express the rational function

$$\frac{x^2+2x-1}{2x^3+3x^2-2x}$$

as a sum of simpler fractions.

(b) **(1pt)** By using the Substitution Rule and knowing that $\int \frac{1}{x}dx = \ln|x| + C$, show that

$$\int \frac{1}{2x-1}dx = \frac{1}{2}\ln|2x-1| + C \quad \text{and} \quad \int \frac{1}{x+2}dx = \ln|x+2| + C.$$

(c) **(0.5pts)** Using (a) and (b), conclude that

$$\int \frac{x^2+2x-1}{2x^3+3x^2-2x}dx = \frac{1}{2}\ln|x| + \frac{1}{10}\ln|2x-1| - \frac{1}{10}\ln|x+2| + C.$$

Solutions

Exercise 1. (a). $3+3+3=9$.

(b). $\int_{-4}^{2} f(x)dx = -3+3-3=-3$.

(c). $\int_{-4}^{2}[f(x)+2x+5]dx = \int_{-4}^{2} f(x)dx + 2\int_{-4}^{2} xdx + 5\int_{-4}^{2} dx = 15$.

Exercise 2. (a). Let $u=x$ and $dv = e^x dx$. Then, $du = dx$ and $v = e^x$. By using Integration by Parts, we have that

$$\int xe^x dx = xe^x - \int e^x dx = xe^x - e^x + C.$$

(b). Indeed, using that $\lim_{t\to-\infty} e^t = \lim_{t\to-\infty} te^t = 0$, we have that

$$\int_{-\infty}^{0} xe^x dx = \lim_{t\to-\infty}\int_{t}^{0} xe^x dx = \lim_{t\to-\infty}[xe^x - e^x]_t^0 = \lim_{t\to-\infty}(-1-te^t+e^t) = -1.$$

Exercise 3. (a). Let $u = 2x+1$. Then, $du = 2dx$ and

$$\int \sqrt{2x+1}dx = \frac{1}{2}\int \sqrt{u}du = \frac{1}{2}\cdot\frac{2}{3}u^{3/2} + C = \frac{1}{3}u^{3/2}+C = \frac{1}{3}(2x+1)^{3/2}+C.$$

(b). Let $u = \sqrt{2x+1} = (2x+1)^{1/2}$. Then,

$$du = \frac{1}{2}(2x+1)^{-\frac{1}{2}}\cdot 2dx = \frac{1}{(2x+1)^{\frac{1}{2}}}dx.$$

This implies that $dx = (2x+1)^{\frac{1}{2}}du = udu$. So, we have that

$$\int \sqrt{2x+1}dx = \int u\cdot udu = \int u^2 du = \frac{1}{3}u^3 + C = \frac{1}{3}(2x+1)^{3/2}+C.$$

Exercise 4. (a). **F**. Indeed, we have that

$$\int_{1}^{\infty}\frac{1}{x}dx = \lim_{t\to\infty}\int_{1}^{t}\frac{1}{x}dx = \lim_{t\to\infty}[\ln(x)]_1^t = \lim_{t\to\infty}[\ln(t)-\ln(1)] = \lim_{t\to\infty}\ln(t) = \infty,$$

which means that $\int_{1}^{\infty}\frac{1}{x}dx$ is divergent.

(b). **T**. Note that the function $f(x) = \dfrac{x^4\tan(x)}{2+\cos(x)}$ is odd and continuous on $\left[-\frac{\pi}{4},\frac{\pi}{4}\right]$. So, $\int_{-\pi/4}^{\pi/4} f(x)dx = 0$ for every a.

(c). **F**. Let $f(x) = x^2$. Then, f is even, continuous on $[-1, 1]$, and $\int_{-1}^{1} f(x)dx = \frac{2}{3} \neq 0$.

(d). **F**. See Exercise 1, items (a) and (b).

Exercise 5. (a). Note first that $2x^3 + 3x^2 - 2x = x(2x^2 + 3x - 2) = x(2x - 1)(x + 2)$. So, if we put

$$\frac{x^2 + 2x - 1}{2x^3 + 3x^2 - 2x} = \frac{A}{x} + \frac{B}{2x - 1} + \frac{C}{x + 2},$$

we get that $A = 1/2$, $B = 1/5$, and $C = -1/10$. Then,

$$\frac{x^2 + 2x - 1}{2x^3 + 3x^2 - 2x} = \frac{1}{2x} + \frac{1}{5}\frac{1}{(2x - 1)} - \frac{1}{10}\frac{1}{(x + 2)}.$$

(b). For the first one, let $u = 2x - 1$. Then, $du = 2dx$ and

$$\int \frac{1}{2x - 1}dx = \frac{1}{2}\int \frac{1}{u}du = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|2x - 1| + C.$$

For the second integral, we put $u = x + 2$ and proceed analogously.

(c). By using (a) and (b), we have

$$\begin{aligned}
\int \frac{x^2 + 2x - 1}{2x^3 + 3x^2 - 2x}dx &= \int \frac{1}{2x}dx + \int \frac{1}{5}\frac{1}{(2x - 1)}dx - \int \frac{1}{10}\frac{1}{(x + 2)}dx \\
&= \frac{1}{2}\int \frac{1}{x}dx + \frac{1}{5}\int \frac{1}{2x - 1}dx - \frac{1}{10}\int \frac{1}{x + 2}dx \\
&= \frac{1}{2}\ln|x| + \frac{1}{10}\ln|2x - 1| - \frac{1}{10}\ln|x + 2| + C.
\end{aligned}$$