

The compact operators and the BPBp

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Joint work with D. García, M. Maestre and M. Martín
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BPBp for compact operators

Bishop-Phelps-Bollobás property

A pair of Banach spaces (X, Y) has the **BPBp** if for every $\varepsilon \in (0, 1)$, there exists $\eta(\varepsilon) > 0$ such that if $T \in S_{\mathcal{L}(X, Y)}$ and $x \in S_X$ satisfy

$$\|T(x)\| > 1 - \eta(\varepsilon),$$

there exist $S \in S_{\mathcal{L}(X, Y)}$ and $x_0 \in S_X$ such that

$$\|S(x_0)\| = 1, \quad \|x_0 - x\| < \varepsilon \quad \text{and} \quad \|T - S\| < \varepsilon.$$

BPBp for compact operators

BPBp for compact operator

A pair of Banach spaces (X, Y) has the **BPBp for compact operators** if for every $\varepsilon \in (0, 1)$, there exists $\eta(\varepsilon) > 0$ such that if $T \in S_{\mathcal{K}(X, Y)}$ and $x \in S_X$ satisfy

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BPBp for compact operators

In the proof of Theorem 2.2 of [JFA, 2008, AAGM] which says that (X, Y) has the BPBp whenever Y has the property β :

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Let $\varepsilon \in (0, 1)$, a norm one **compact operator** T and a norm one vector x_0 satisfying

$$\|T(x_0)\| > 1 - \frac{\varepsilon^2}{4}.$$

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The bounded linear operator $S : X \rightarrow Y$ defined by

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is also **compact** and satisfies

$$\|S(z_0)\| = \|S\| = 1, \quad \|z_0 - x_0\| < \varepsilon \quad \text{and} \quad \|S - T\| < \varepsilon.$$

for some $z_0 \in S_X$.

BPBp for compact operators

More examples

BPBp for compact operators

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- X is uniformly convex and Y is any Banach space.
[2014, *Canad. J. Math.*, S. K. Kim and H. J. Lee]

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[2014, *JFA*, Y. S. Choi, S. K. Kim, H. J. Lee and M. Martín]
- $X = L_1(\mu)$ and $Y = L_\infty(\nu)$ for any measure μ and any localizable measure ν
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[2014, *JFA*, Y. S. Choi, S. K. Kim, H. J. Lee and M. Martín]
- $X = C(K_1)$ and $Y = C(K_2)$ in the real case
[2014, *Nonlinear Anal.*, S. K. Kim, H. J. Lee, et al.,]

BPBp for compact operators

More examples

- $X = C_0(L)$ and Y uniformly convex for L a locally compact Hausdorff topological space
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- X is an arbitrary Banach space and Y^* is isometrically isomorphic to a $L_1(\mu)$ -space
[2014, *Nonlinear Anal.*, S. K. Kim, H. J. Lee, et al.,]
- $X = L_1(\mu)$ for an arbitrary measure μ and Y having the AHSP
[2014, *JMAA*, Acosta, Becerra-Guerrero, García, Kim and Maestre]

BPBp for compact operators

Questions:

- (1) BPBp for compact operators \Rightarrow BPBp?

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- (2) BPBp \Rightarrow BPBp for compact operators?

Answers:

- (1) $(L_1[0, 1], C[0, 1])$ has the BPBp for compact operators **BUT** $NA(L_1[0, 1], C[0, 1])$ is not dense in $\mathcal{L}(L_1[0, 1], C[0, 1])$.

BPBp for compact operators

Questions:

- (1) BPBp for compact operators \Rightarrow BPBp?
- (2) BPBp \Rightarrow BPBp for compact operators?

Answers:

- (1) $(L_1[0, 1], C[0, 1])$ has the BPBp for compact operators **BUT** $NA(L_1[0, 1], C[0, 1])$ is not dense in $\mathcal{L}(L_1[0, 1], C[0, 1])$.
- (2) Open question.

BPBp for compact operators

Martín's counterexample:

BPBp for compact operators

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[JFA, 2014, M. Martín, Norm attaining compact operators]

There exist compact operators which cannot be approximated by norm attaining ones.

BPBp for compact operators

Martín's counterexample:

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There exist compact operators which cannot be approximated by norm attaining ones.

The counterexample uses the 1973 Enflo's counterexample which shows that there exists a closed subspace of c_0 without the approximation property.

BPBp for compact operators

- In [JFA, 2014, M. Martín, Norm attaining compact operators], the author studied the conditions that X and Y must have in order to approximate compact operators by norm attaining ones.

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- In [JFA, 2014, M. Martín, Norm attaining compact operators], the author studied the conditions that X and Y must have in order to approximate compact operators by norm attaining ones.
- We did the same thing but now considering the BPBp for compact operators.

BPBp for compact operators

Technical Lemma (domain space)

Let X and Y be Banach spaces. Suppose that there exists a function $\eta : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ such that given $\delta > 0$, $x_1^*, \dots, x_n^* \in B_{X^*}$ and $x_0 \in S_X$,

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$$(i) \quad \|P^* x_j^* - x_j^*\| < \delta, \text{ for all } j = 1, \dots, n$$

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- (i) $\|P^* x_j^* - x_j^*\| < \delta$, for all $j = 1, \dots, n$
- (ii) $\|i(P(x_0)) - x_0\| < \delta$,

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- (ii) $\|i(P(x_0)) - x_0\| < \delta$,
- (iii) $P \circ i = Id_{P(X)}$,
- (iv) the pair $(P(X), Y)$ has the BPBp for compact operators with the function $\eta(\varepsilon)$.

BPBp for compact operators

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- (ii) $\|i(P(x_0)) - x_0\| < \delta$,
- (iii) $P \circ i = Id_{P(X)}$,
- (iv) the pair $(P(X), Y)$ has the BPBp for compact operators with the function $\eta(\varepsilon)$.

Then the pair (X, Y) has the BPBp for compact operators.

BPBp for compact operators

Example: $X = C_0(L)$ satisfies the hypothesis of this lemma.

BPBp for compact operators

Example: $X = C_0(L)$ satisfies the hypothesis of this lemma.

Theorem

Let L be a locally compact Hausdorff topological space and let Y be a Banach space. If (c_0, Y) has the BPBp for compact operators, then $(C_0(L), Y)$ has the BPBp for compact operators.

BPBp for compact operators

Corollary

Let L be a locally compact Hausdorff topological space and let Y be a Banach space. Suppose that there exist a set I , $\{y_i : i \in I\} \subset S_Y$, $\{y_i^* : i \in I\} \subset S_{Y^*}$, a subset $E \subset S_Y$, a mapping $F : E \rightarrow S_{Y^*}$ and $0 \leq \rho < 1$ satisfying that

- (1) $y_i^*(y_i) = 1, \forall i \in I$;
- (2) $|y_i^*(y_j)| \leq \rho, \forall i, j \in I, i \neq j$;
- (3) E is uniformly strongly exposed by F ;
- (4) $|F(e)(y_i)| \leq \rho, \forall e \in E, i \in I$;
- (5) for any $y \in Y$,

$$\|y\| = \max\{\sup\{|y_i^*(y)| : i \in I\}, \sup\{|F(e)(y)| : e \in E\}\}.$$

Then, the pair $(C_0(L), Y)$ has the BPBp for compact operators.

BPBp for compact operators

Lemma

Let X, Y be Banach spaces. Then the following are equivalent:

BPBp for compact operators

Lemma

Let X, Y be Banach spaces. Then the following are equivalent:

- (i) The pair $(c_0(X), Y)$ has the BPBp for compact operators;
- (ii) there is a function $\eta : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ such that the pairs $(\ell_\infty^m(X), Y)$ with $m \in \mathbb{N}$ have the BPBp for compact operators with the function η .

BPBp for compact operators

Lemma

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Moreover, when $\mathcal{K}(X, Y) = \mathcal{L}(X, Y)$ (in particular, if one of the spaces X or Y is finite-dimensional), this happens when $(c_0(X), Y)$ or $(\ell_\infty(X), Y)$ has the BPBp.

BPBp for compact operators

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Moreover, when $\mathcal{K}(X, Y) = \mathcal{L}(X, Y)$ (in particular, if one of the spaces X or Y is finite-dimensional), this happens when $(c_0(X), Y)$ or $(\ell_\infty(X), Y)$ has the BPBp.

Corollary

Let Y be a Banach space. If the pair (c_0, Y) has the BPBp, then it has the BPBp for compact operators.

BPBp for compact operators

Technical lemma

Let X be Banach space for which there exists a net $\{P_\alpha\}_{\alpha \in \Lambda}$ of rank-one projections on X such that

BPBp for compact operators

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Let X be Banach space for which there exists a net $\{P_\alpha\}_{\alpha \in \Lambda}$ of rank-one projections on X such that

- (i) $\{P_\alpha x\} \rightarrow x$ for all $x \in X$ in norm and

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If for a Banach space Y there exists a function $\eta : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ such that all the pairs $(P_\alpha(X), Y)$ with $\alpha \in \Lambda$ have the BPBp for compact operators with the function η , then the pair (X, Y) has the BPBp for compact operators.

BPBp for compact operators

Theorem

Let μ be a positive measure, let X be a Banach space such that X^* has the Radon-Nikodým property and let Y be a Banach space. If $(\ell_1(X), Y)$ has the BPBp for compact operators, then the pair $(L_1(\mu, X), Y)$ has the BPBp for compact operators.

BPBp for compact operators

Corollary

Let μ be a positive measure and let X, Y be Banach spaces. The pair $(L_1(\mu, X), Y)$ has the BPBp for compact operators in the following cases:

- (a) if X and Y are finite-dimensional;

BPBp for compact operators

Corollary

Let μ be a positive measure and let X, Y be Banach spaces. The pair $(L_1(\mu, X), Y)$ has the BPBp for compact operators in the following cases:

- (a) if X and Y are finite-dimensional;
- (b) if X^* has the Radon-Nikodým property, Y is a Hilbert space and the pair (X, Y) has the BPBp for compact operators;

BPBp for compact operators

Corollary

Let μ be a positive measure and let X, Y be Banach spaces. The pair $(L_1(\mu, X), Y)$ has the BPBp for compact operators in the following cases:

- (a) if X and Y are finite-dimensional;
- (b) if X^* has the Radon-Nikodým property, Y is a Hilbert space and the pair (X, Y) has the BPBp for compact operators;
- (c) in particular, if Y is a Hilbert space and $X = c_0$ or $X = L_p(\nu)$ for any positive measure ν and $1 < p < \infty$.

BPBp for compact operators

Auxiliary result

Let Y be a Banach space. Then, the following are equivalent:

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BPBp for compact operators

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Let Y be a Banach space. Then, the following are equivalent:

- (i) the pair (ℓ_1, Y) has the BPBp for compact operators;
- (ii) Y has the AHSP;

BPBp for compact operators

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Let Y be a Banach space. Then, the following are equivalent:

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BPBp for compact operators

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- (i) the pair (ℓ_1, Y) has the BPBp for compact operators;
- (ii) Y has the AHSP;
- (iii) the pair (ℓ_1, Y) has the BPBp;
- (iv) for every positive measure μ , the pair $(L_1(\mu), Y)$ has the BPBp for compact operators;

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- (i) the pair (ℓ_1, Y) has the BPBp for compact operators;
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- (iii) the pair (ℓ_1, Y) has the BPBp;
- (iv) for every positive measure μ , the pair $(L_1(\mu), Y)$ has the BPBp for compact operators;
- (v) there is a positive measure μ such that $L_1(\mu)$ is infinite-dimensional and the pair $(L_1(\mu), Y)$ has the BPBp for compact operators.

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BPBp for compact operators

Technical lemma

Let X be a Banach space. Let $\{P_\alpha\}_{\alpha \in \Lambda}$ be a net of norm-one projections on X such that

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- (ii) $\{P_\alpha^* x^*\} \rightarrow x^*$ in norm for all $x^* \in X^*$.

BPBp for compact operators

Technical lemma

Let X be a Banach space. Let $\{P_\alpha\}_{\alpha \in \Lambda}$ be a net of norm-one projections on X such that

- (i) $\alpha \preceq \beta$ implies that $P_\alpha(X) \subset P_\beta(X)$ and
- (ii) $\{P_\alpha^* x^*\} \rightarrow x^*$ in norm for all $x^* \in X^*$.

If for a Banach space Y there exists a function $\eta : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ such that all the pairs $(P_\alpha(X), Y)$ with $\alpha \in \Lambda$ have the BPBp for compact operators with the function η , then the pair (X, Y) has the BPBp for compact operators.

BPBp for compact operators

Theorem

Let X be a Banach space such that X^* is isometrically isomorphic to ℓ_1 and let Y be a Banach space. If the pair (c_0, Y) has the BPBp for compact operators, then (X, Y) has the BPBp for compact operators.

BPBp for compact operators

Corollary

Let X be a Banach space such that X^* is isometrically isomorphic to ℓ_1 and let Y be a Banach space. Suppose that there exist a set I , $\{y_i : i \in I\} \subset S_Y$, $\{y_i^* : i \in I\} \subset S_{Y^*}$, a subset $E \subset S_Y$, a mapping $F : E \rightarrow S_{Y^*}$ and $0 \leq \rho < 1$ satisfying that

- (1) $y_i^*(y_i) = 1, \forall i \in I$;
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- (4) for any $y \in Y$,

$$\|y\| = \max\{\sup\{|y_i^*(y)| : i \in I\}, \sup\{|F(e)(y)| : e \in E\}\}.$$

Then, the pair (X, Y) has the BPBp for compact operators.

BPBp for compact operators

Technical lemma (range space)

Let X and Y be Banach spaces. Suppose that there exists a net of **norm-one** projections $\{Q_\lambda\}_{\lambda \in \Lambda} \subset \mathcal{L}(X, Y)$ such that

$$\{Q_\lambda y\} \longrightarrow y \text{ in norm for every } y \in Y.$$

If there is a function $\eta : \mathbb{R}^+ \longrightarrow \mathbb{R}^+$ such that the pairs $(X, Q_\lambda(Y))$ with $\lambda \in \Lambda$ have the BPBp for compact operators with the function η , then the pair (X, Y) has the BPBp for compact operators.

BPBp for compact operators

Theorem

Let X, Y be Banach spaces.

- (a) For $1 \leq p < \infty$, if the pair $(X, \ell_p(Y))$ has the BPBp for compact operators, then so does $(X, L_p(\mu, Y))$ for every positive measure μ such that $L_1(\mu)$ is infinite-dimensional.

BPBp for compact operators

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- (b) If the pair (X, Y) has the BPBp for compact operators, then so does $(X, L_\infty(\mu, Y))$ for every σ -finite positive measure μ .

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Let X, Y be Banach spaces.

- (a) For $1 \leq p < \infty$, if the pair $(X, \ell_p(Y))$ has the BPBp for compact operators, then so does $(X, L_p(\mu, Y))$ for every positive measure μ such that $L_1(\mu)$ is infinite-dimensional.
- (b) If the pair (X, Y) has the BPBp for compact operators, then so does $(X, L_\infty(\mu, Y))$ for every σ -finite positive measure μ .
- (c) If the pair (X, Y) has the BPBp for compact operators, then so does $(X, C(K, Y))$ for every compact Hausdorff topological space K .

BPBp for compact operators

Corollary

Let X, Y be Banach spaces, let K be a compact Hausdorff topological space, let μ be a positive measure and let ν be a σ -finite positive measure.

- (a) If Y has property β , then $(X, L_\infty(\mu, Y))$ and $(X, C(K, Y))$ have the BPBp for compact operators.

BPBp for compact operators

Corollary

Let X, Y be Banach spaces, let K be a compact Hausdorff topological space, let μ be a positive measure and let ν be a σ -finite positive measure.

- (a) If Y has property β , then $(X, L_\infty(\mu, Y))$ and $(X, C(K, Y))$ have the BPBp for compact operators.
- (b) If Y has the AHSP, then so do $L_\infty(\nu, Y)$ and $C(K, Y)$.

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Corollary

Let X, Y be Banach spaces, let K be a compact Hausdorff topological space, let μ be a positive measure and let ν be a σ -finite positive measure.

- (a) If Y has property β , then $(X, L_\infty(\mu, Y))$ and $(X, C(K, Y))$ have the BPBp for compact operators.
- (b) If Y has the AHSP, then so do $L_\infty(\nu, Y)$ and $C(K, Y)$.
- (c) For $1 \leq p < \infty$, if $\ell_p(Y)$ has the AHSP and $L_1(\mu)$ is infinite-dimensional, then so does $L_p(\mu, Y)$.

BPBp for compact operators

Thank you very much
for your attention!