Calculus 1 (BE5B01MA1)
Lab 1

In all the following exercises, students are encouraged to draw the intervals and graphs of the functions in order to see what happens geometrically.

Exercise 1. Decide if the following claims are true (T) or false (F) by justifying your answer.

(a) If $x < 2$, then $x^2 < 4$. (F)

(b) If $x < 2$, then $x \leqslant 3$. (T)

(c) If $x^2 < 4$, then $x < 2$. (T)

(d) If $x = 3$, then $x \leqslant 3$. (T)

(e) If $0 \leqslant x \leqslant 2$, then $x^2 \leqslant 4$. (T)

(f) If $|x| > 2$, then $x > 2$. (F)

Exercise 2. Find the domain of the following functions

(a) $f(x) = 3x$. $D(f) = \mathbb{R}$

(b) $g(x) = -x$. $D(f) = \mathbb{R}$

(c) $h(x) = |x - 1|$. $D(f) = \mathbb{R}$

(d) $f(x) = \dfrac{|2x-1|}{2x+1}$. $D(f) = \mathbb{R} \setminus \{-\frac{1}{2}\}$

(e) $g(x) = \dfrac{x^2-1}{x+1}$. $D(f) = \mathbb{R} \setminus \{-1\}$

(f) $h(x) = \dfrac{|x|}{x}$. $D(f) = \mathbb{R} \setminus \{0\}$

Exercise 3. Find the domain of the following functions

(a) $f(x) = \sqrt{x(2-3x)}$. $D(f) = [0, 2/3]$

(b) $g(x) = \sqrt{\dfrac{2x-1}{1-3x}}$. $D(f) = (1/3, 1/2]$

(c) $h(x) = \sqrt{x} - \sqrt{5-2x}$. $D(f) = [0, 5/2]$

(d) $k(x) = e^x + \dfrac{x+3}{x^2-1}$. $D(f) = \mathbb{R} \setminus \{-1, 1\}$

Exercise 4. Verify that

$$\sqrt{1+x^2} - |x| = \frac{1}{\sqrt{1+x^2} + |x|}$$

and conclude that when x grows, the value of the function $\sqrt{1+x^2} - |x|$ is close to zero.

Exercise 5. Decide if the following functions are even or odd.

(a) $f(x) = x^3$. (odd)

(b) $g(x) = x^2$. (even)

(c) $h(x) = 2x - x^2$. (neither even non odd)

(d) $k(x) = |x|$. (even)

Exercise 6. Show that $Im(f) \subset D(f)$ and, then, calculate $g \circ f$.

(a) $f(x) = -\sqrt{x}$ and $g(x) = \sqrt{2-x}$.

(b) $f(x) = \dfrac{x}{x+1}$ and $g(x) = \dfrac{x+1}{x-1}$.

Exercise 7 (Group discussion)**.** Find expressions for the quadratic functions whose graphs are shown.