Calculus 1 (BE5B01MA1)
Lab 10

Exercise 1. Determine whether each of the following integrals is convergent or divergent. Evaluate those that are convergent.

(1) $\displaystyle\int_3^{\infty} \frac{1}{(x-2)^{3/2}}\,dx$

(2) $\displaystyle\int_0^{\infty} \frac{1}{\sqrt[4]{1+x}}\,dx$

(3) $\displaystyle\int_{-\infty}^{0} \frac{1}{3-4x}\,dx$

(4) $\displaystyle\int_1^{\infty} \frac{1}{(2x+1)^3}\,dx$

(5) $\displaystyle\int_2^{\infty} e^{-5x}\,dx$

(6) $\displaystyle\int_0^{\infty} \frac{x^2}{\sqrt{1+x^3}}\,dx$

(7) $\displaystyle\int_{-\infty}^{\infty} (x^3-3x^2)\,dx.$

(8) $\displaystyle\int_{-\infty}^{\infty} xe^{-x^2}\,dx$

(9) $\displaystyle\int_1^{\infty} \frac{\ln(x)}{x^3}\,dx$

(10) $\displaystyle\int_{-\infty}^{\infty} \cos(\pi x)\,dx$

(11) $\displaystyle\int_1^{\infty} \frac{\ln(x)}{x}\,dx$

(12) $\displaystyle\int_0^{\infty} \frac{e^x}{e^{2x}+3}\,dx.$

Exercise 2. Explain why each of the following integrals is improper, decide if they are convergent or divergent, and finally evaluate those that are convergent.

(1) $\displaystyle\int_0^{9} \frac{1}{\sqrt[3]{x-1}}\,dx$

(2) $\displaystyle\int_0^{5} \frac{x}{x-2}\,dx$

(3) $\displaystyle\int_{-1}^{1} \frac{e^x}{e^x-1}\,dx.$

(4) $\displaystyle\int_0^{1} \frac{\ln(x)}{\sqrt{x}}\,dx$

Exercise 3. Sketch the region enclosed by the given curves. Then, find the area of the region.

(1) $y=e^x$, $y=x^2-1$,$x=-1$, and $x=1$.

(2) $y=\sin(x)$, $y=x$, $x=\pi/2$, and $x=\pi$.

(3) $y=(x-2)^2$ and $y=x$.

(4) $x=1-y^2$ and $x=y^2-1$.

(5) $4x+y^2=12$ and $x=y$.

(6) $y=\cos(x)$ and $y=2-\cos(x)$ for $0\leqslant x\leqslant 2\pi$.

(7) $y=|x|$ and $y=x^2-2$.

(8) $y=\dfrac{1}{x}$, $y=x$, and $y=\dfrac{1}{4}x$ for $x>0$.

Exercise 4 (Group discussion)**.**

(a) Show that $\int_{-\infty}^{\infty} x dx$ is divergent.

(b) Show that $\lim_{t\to\infty} \int_{-t}^{t} x dx = 0$.

(c) Can we define **in general** the value of $\int_{-\infty}^{\infty} f(x)dx$ as $\lim_{t\to\infty} \int_{-t}^{t} f(x)dx$?

Exercise 5 (Group discussion)**.** Consider the following integral:

$$\int_{0}^{\infty} \frac{1}{\sqrt{x}(1+x)} dx.$$

(a) Explain why this integral is improper.

(b) Evaluate it by expressing it as a sum of improper integrals of type 2 and 1.