Calculus 1 (BE5B01MA1)
Lab 12

Exercise 1 (Telescopic and geometric series). Show that each one of the following series are either telescopic or geometric and calculate its sum.

(1) $\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^n$

(2) $\sum_{n=1}^{\infty}4\left(\frac{2}{5}\right)^n$

(3) $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n+1}\cdot\sqrt{n}(\sqrt{n+1}+\sqrt{n})}$

(4) $\sum_{n=1}^{\infty}\frac{2n+1}{n^2(n+1)^2}$

(5) $\sum_{n=1}^{\infty}\frac{4}{4n^2+4n-3}$

(6) $\sum_{n=1}^{\infty}\left(\frac{1}{2^{n-2}}-\frac{1}{3^{n+2}}\right)$

(7) $\sum_{n=1}^{\infty}\frac{3}{9n^2+3n-2}$

(8) $\sum_{n=1}^{\infty}\frac{1}{4n^2-1}$

(9) $\sum_{n=1}^{\infty}\frac{2}{(4n-3)(4n+1)}$

(10) $\sum_{n=1}^{\infty}\ln\left[\frac{(n+1)^2}{n(n+2)}\right]$

Exercise 2 (Limit test). Show that the following series are all divergent.

(1) $\sum_{n=1}^{\infty}(\sqrt{n}+\sqrt{n+1})$

(2) $\sum_{n=1}^{\infty}[1+(-1)^n]$

(3) $\sum_{n=1}^{\infty}\frac{n^3}{n^3+n^2+4}$

(4) $\sum_{n=1}^{\infty}\frac{n}{\cos(n)}$

(5) $\sum_{n=1}^{\infty}n\sin\left(\frac{1}{n}\right)$

(6) $\sum_{n=1}^{\infty}\frac{n!}{2^n}$

Exercise 3 (Comparison Test). Determine if the following series are convergent or divergent.

(1) $\sum_{n=1}^{\infty}\frac{1}{n^4+n^2+1}$

(2) $\sum_{n=1}^{\infty}\frac{1}{n3^n}$

(3) $\sum_{n=1}^{\infty}\frac{\sqrt{n}}{n^2+1}$

(4) $\sum_{n=1}^{\infty}\frac{2+\cos(n)}{n^2}$

(5) $\sum_{n=1}^{\infty}\frac{1}{(n-1)^2}$

(6) $\sum_{n=1}^{\infty}\frac{\ln(n)}{n^3}$

(7) $\sum_{n=1}^{\infty}\frac{n+5}{n2^n}$

(8) $\sum_{n=1}^{\infty}\frac{1+2^n}{1+3^n}$

Exercise 4 (Integral test)**.** Determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{1}{n^5}, \quad \sum_{n=1}^{\infty} \frac{1}{\sqrt{n+4}}, \quad \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}.$$

Exercise 5 (The root test)**.** Verify that the following series are convergent.

$$\sum_{n=1}^{\infty} \left(\frac{-n}{3n+1}\right)^n, \quad \sum_{n=1}^{\infty} (\sqrt[n]{n}-1)^n, \quad \sum_{n=1}^{\infty} \frac{n^5}{5^n}, \quad \sum_{n=1}^{\infty} \frac{(-5)^{n+1}}{(\ln(n))^n}.$$

Exercise 6. Decide which test is more convenient and show whether the following series are convergent or divergent.

(1) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^n}$

(2) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{n!}$

(3) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{n!}$

(4) $\displaystyle\sum_{n=1}^{\infty} \frac{(2^n+3^n)^{1/n}}{n}$

(5) $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$

(6) $\displaystyle\sum_{n=1}^{\infty} \frac{3n}{\sqrt{n^3+1}}$

(7) $\displaystyle\sum_{n=1}^{\infty} \frac{n3^{2n}}{5^{n-1}}$

(8) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \cos(n)}{n^2}$

(9) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+2}$

(10) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(\ln(n))^n}$

(11) $\displaystyle\sum_{n=1}^{\infty} n!\left(\frac{2}{n}\right)^n$

(12) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n(n+2)}$

Exercise 7. For which of the following series if the Ratio Test inconclusive?

$$\sum_{n=1}^{\infty} \frac{1}{n^3}, \qquad \sum_{n=1}^{\infty} \frac{n}{2^n}, \qquad \sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{\sqrt{n}}, \qquad \sum_{n=1}^{\infty} \frac{\sqrt{n}}{1+n^2}.$$

Exercise 8. For what values of p is the following series convergent?

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^p}$$

Exercise 9. Decide if the following series are convergent.

$$\sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdot \ldots \cdot (2n-1)}{n!}, \quad \sum_{n=1}^{\infty} \frac{2 \cdot 4 \cdot 6 \cdot \ldots \cdot (2n)}{1 \cdot 4 \cdot 7 \cdot \ldots \cdot (3n-2)}, \quad \sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdot \ldots \cdot (2n-1)}{n! 2^n}.$$

Exercise 10. For which positive values of k is the following series convergent?

$$\sum_{n=1}^{\infty} \frac{(n!)^2}{(kn)!}$$

Exercise 11 (Group discussion)**.** True **(T)** or false **(F)**?

( ) If $\lim_{n\to\infty} a_n = 0$, then $\sum a_n$ converges.

( ) If $\sum a_n$ diverges, then $\lim_{n\to\infty} a_n \neq 0$.

( ) If $\sum a_n$ converges and $a_n \geqslant 0$ for all $n \in \mathbb{N}$, then $\sum \sqrt{a_n}$ converges.

( ) If $\sum a_n$ diverges, then $\sum_{n=1}^{\infty} a_n^2$ diverges.

( ) If $\sum a_n$ and $\sum b_n$ diverge, then $\sum (a_n + b_n)$ diverges.

( ) If $\sum a_n$ diverges and $a_n \neq 0$ for all $n \in \mathbb{N}$, then $\sum \frac{1}{a_n}$ diverges.

( ) If $\sum a_n$ converges, then $\sum_{n=100}^{\infty} a_n$ also converges.

( ) If $a_n > 0$ for all $n \in \mathbb{N}$ and $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 1$, then $\sum a_n$ is divergent.

( ) If $(a_n)_{n=1}^{\infty}$ is a constant sequence, then $\sum a_n$ is convergent.