CALCULUS 1 (BE5B01MA1)
LAB 8

Exercise 1 (Substitution Rule)**.** Evaluate the indefinite integrals.

(1) $\int x\sin(x^2)dx.$

(2) $\int (1-2x)^9 dx.$

(3) $\int (x+1)\sqrt{2x+x^2}dx.$

(4) $\int \sec(3\theta)\tan(3\theta)d\theta.$

(5) $\int \frac{a+bx^2}{\sqrt{3ax+bx^3}}dx$

(6) $\int \sec^2(\theta)\tan^3(\theta)d\theta.$

(7) $\int (x^2+1)(x^3+3x)^4 dx.$

(8) $\int \frac{\cos(x)}{\sin^2(x)}dx.$

(9) $\int \sqrt{\cot(x)}\csc^2(x)dx.$

(10) $\int \frac{z^2}{\sqrt[3]{1+z^3}}dz.$

(11) $\int x(2x+5)^8 dx.$

(12) $\int \sqrt[3]{1+7x}dx.$

Exercise 2 (Substitution Rule)**.** Evaluate the definite integrals.

(1) $\int_1^2 (8x^3+3x^2)dx.$

(2) $\int_0^1 (1-x^9)dx.$

(3) $\int_1^9 \frac{\sqrt{u}-2u^2}{u}du.$

(4) $\int_0^1 y(y^2+1)^5 dy$

(5) $\int_0^1 v^2\cos(v^3)dv$

(6) $\int_{-\pi/4}^{\pi/4} \frac{t^4\tan(t)}{2+\cos(t)}dt.$

(7) $\int_0^{\pi/8} \sec(2\theta)\tan(2\theta)d\theta.$

(8) $\int_0^T (x^4-8x+7)dx.$

(9) $\int_{-1}^1 \frac{\sin(x)}{1+x^2}dx.$

(10) $\int_0^1 \sin(3\pi t)dt.$

The red means that these are not by substitution!!! Just sum/difference rule.

Exercise 3. Let f be a continuous function.

(a) If $\int_0^9 f(x)dx = 4$, find $\int_0^3 xf(x^2)dx.$

(b) If $\int_0^4 f(x)dx = 10$, find $\int_0^2 f(2x)dx.$

Exercise 4. Suppose that a and b are positive numbers. Show that

$$\int_0^1 x^a(1-x)^b dx = \int_0^1 x^b(1-x)^a dx.$$

Exercise 5 (Integration by parts)**.** Evaluate the integrals.

(1) $\displaystyle\int x\cos(5x)dx.$

(2) $\displaystyle\int xe^{-3x}dx.$

(3) $\displaystyle\int (x^2+2x)\cos(x)dx.$

(4) $\displaystyle\int \ln(2x+1)dx.$

(5) $\displaystyle\int e^{2x}\sin(3x)dx$

(6) $\displaystyle\int \frac{xe^{2x}}{(1+2x)^2}dx.$

(7) $\displaystyle\int_0^{1/2} x\cos(\pi x)dx.$

(8) $\displaystyle\int_1^3 x^3\ln(x)dx.$

(9) $\displaystyle\int_0^1 (x^2+1)e^{-x}dx.$

(10) $\displaystyle\int_4^9 \frac{\ln(x)}{\sqrt{x}}dx.$

(11) $\displaystyle\int_0^1 \frac{x^3}{\sqrt{4+x^2}}dx.$

(12) $\displaystyle\int_1^2 (\ln(x))^2dx.$

Exercise 6. Prove the reduction formula: for $n \geqslant 2$ integer, we have

$$\int \sin^n(x)dx = -\frac{1}{n}\cos(x)\sin^{n-1}(x) + \frac{n-1}{n}\sin^{n-2}(x)dx.$$

Exercise 7. First make a substitution and then use integration by parts to evaluate the integral.

(1) $\displaystyle\int \cos(\sqrt{x})dx.$

(2) $\displaystyle\int_{\sqrt{\pi/2}}^{\sqrt{\pi}} x^3\cos(x^2)dx.$

(3) $\displaystyle\int x^3e^{-x^2}dx.$

(4) $\displaystyle\int_1^4 e^{\sqrt{x}}dx.$

Exercise 8 (Group discussion)**.** True **(T)** or false **(F)**? If it is (T), explain why. If it is (F), give a counterexample.

(   ) If f and g are continuous on $[a,b]$, then $\displaystyle\int_a^b [f(x)g(x)]dx = \left(\int_a^b f(x)dx\right)\left(\int_a^b g(x)dx\right).$

(   ) If f and g are continuous on $[a,b]$, then $\displaystyle\int_a^b xf(x)dx = x\int_a^b f(x)dx.$

(   ) If f is continuous on $[a,b]$ and $f(x)\geqslant 0$, then $\displaystyle\int_a^b \sqrt{f(x)}dx = \sqrt{\int_a^b f(x)dx}.$

( ) $\int_{-5}^{5}(ax^2+bx+c)dx = 2\int_{0}^{5}(ax^2+c)dx.$

( ) If $\int_{0}^{1} f(x)dx = 0$, then $f(x) = 0$ for $0 \leqslant x \leqslant 1$.