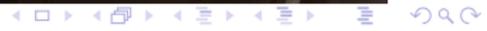# On holomorphic functions attaining their weighted norms

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Departament d'Anàlisi Matemàtica

18th ILJU School of Mathematics
Jan 30th - Feb 3rd 2023, Seoul, South Korea

매우 감사합니다!

생일 축하해!

Based on a joint work with

⋆ Rubén Medina
  (Granada University and Czech Technical University in Prague)

Goals

⋆ Holomorphic functions

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* ★ Holomorphic functions

* ★ Weighted norms

Goals

* $\star$ Holomorphic functions

* $\star$ Weighted norms

* $\star$ Connection between the classical norm-attaining theory

Literature and relevance

- Weighted and Bloch spaces of holomorphic functions
  - ⋆ [2021, J. Bonet, M. Lindström, E. Wolf]
  - ⋆ [2012, M.J. Beltrán-Meneu]
  - ⋆ [2013, E. Jordá]
  - ⋆ [2014, O. Blasco, P. Galindo, A. Miralles]
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- They are natural objects in
  - ⋆ Partial differential equations
  - ⋆ Complex Analysis
  - ⋆ Operator theory
  - ⋆ Spectral theory

- When the supremum is attained in this context?

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    ⋆ [1996, Y.S. Choi and G.S. Kim]
    ⋆ [2004, M.D. Acosta, J. Alaminos, D. García, and M. Maestre]
    ⋆ [2015, D. Carando and M. Mazzitelli]
    ⋆ [2019, S.K. Kim and H.J. Lee]
    ⋆ [2022, M. Jung]

*When the weighted norm
of a holomorphic function is attained?*

Main problem

$\star$ Infinite dimensional setting.

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    * [1998, A. Montes-Rodríguez]
    * [2010, M.J. Martín]
    * [2012, J. Bonet, M. Lindström, E. Wolf]

Notation

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- ⋆ $\mathcal{A}_u(B_X; Y) \leq H_v(B_X; Y) =$ uniformly continuous on B_X

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⋆ $s = 1 \Rightarrow \|\cdot\|_\infty$.

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Our setting

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Dotting the i's and crossing the t's...

Norm-attaining elements

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$\star$ Maximum Modulus Principle $\Rightarrow$ f attains its s-norm on sS_X.
  [1986, Mujica]

$\star$ Connection?

* Connection?
  * Functions attaining their weighted norms
  * Functions attaining their supremum norms on smaller balls.

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TFAE:

* f attains its v-norm.
* $\exists\ s_0 \in (0,1)$ with $\|f\|_v = (1 - s_0^2)\|f\|_{s_0}$ and f attains its s_0-norm.

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⋆ We say that $P : X \to Y$ is a **polynomial of degree at most** N if there exists $P_k \in \mathcal{P}(^kX; Y)$, $k = 1, \ldots, N$ such that

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  [Mujica, Complex Analysis in Banach spaces, 1956]
  [P. Hájek, M. Johanis, Smooth Analysis in Banach spaces, 2014]
  [Defant, García, Maestre, Sevilla-Peris, 2019]

Relation between
the weighted and supremum norms

★ Complete study of the relation between v- and s-norms on $\mathcal{P}(^N X; Y)$.

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[2022, D., R. Medina]

For every $N \in \mathbb{N}$, there is $\delta_N \in (0,1)$ with $\delta_N \to 0$ as $N \to \infty$ such that, for every $s \in (0,1]$ and for every $P \in \mathcal{P}(^N X; Y)$

Relation between $\|\cdot\|_v$ and $\|\cdot\|_\infty$

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$$\|P\|_v = \frac{\delta_N}{s^N} \cdot \|P\|_s.$$

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> $$\|P\|_v = \frac{\delta_N}{s^N} \cdot \|P\|_s.$$

Proof:

$$\sup_{x \in B_X} (1 - \|x\|^2) \|P(x)\|$$

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Proof:

$$\sup_{x \in B_X}(1 - \|x\|^2)\|P(x)\| = \sup_{x \in B_X}(1 - \|x\|^2)\left(\frac{\|x\|}{s}\right)^N \left\|P\left(s\frac{x}{\|x\|}\right)\right\|$$

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&= \sup_{r \in [0,1]} (r^N - r^{N+2})\frac{1}{s^N}\,\|P\|_s.
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Moreover, TFAE:

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Moreover, TFAE:

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★ P attains its s-norm for every $s \in (0,1]$.

$\star$ A basic but relevant result.

Relation between $\|\cdot\|_v$ and $\|\cdot\|_\infty$

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[2022, D., R. Medina]

Let X be a reflexive space. Every weakly sequentially continuous function $f : X \longrightarrow \mathbb{C}$ bounded on B_X attains its s-norm for every $s \in (0,1]$. In particular, f attains its v-norm.

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* $f : X \to \mathbb{C}$ be weakly sequentially continuous and $s \in (0,1]$
* $(x_n)_n \subseteq sS_X$ such that $|f(x_n)| \longrightarrow \|f\|_s$

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⋆ $|f(x_0)| = \|f\|_s$

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$\star$ Therefore, f attains its v-norm

Relation between $\|\cdot\|_v$ and $\|\cdot\|_\infty$

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Let X be a reflexive space. Suppose that $\mathcal{P}^N(X; Y) = \mathcal{P}^N_{wsc}(X; Y)$. Then,

$$\mathrm{NA}_v \, \mathcal{P}^N(X; Y) = \mathcal{P}^N(X; Y) = \mathrm{NA} \, \mathcal{P}^N(X; Y).$$

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⋆ It **does not** hold if one of the hypothesis is dropped.

[2022, M. Jung]

Let X be a reflexive space. Suppose that $\mathcal{P}^N(X;Y) = \mathcal{P}^N_{wsc}(X;Y)$. Then,

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    [2022, D. R. Medina]

Relation between $\|\cdot\|_v$ and $\|\cdot\|_\infty$

[2022, D., R. Medina]

For every $N \in \mathbb{N}$, there is $\delta_N \in (0,1)$ with $\delta_N \to 0$ as $N \to 0$ such that, for every $s \in (0,1]$ and for every $P \in \mathcal{P}(^N X; Y)$

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[1996, Y.S. Choi, G.S. Kim]
[2004, M.D. Acosta, J. Alaminos, D. García, M. Maestre]
[2022, M. Jung]

Non-homogeneous polynomials

$\star$ We are "forced" to move forwards to $\mathcal{P}^N(X;Y)$

- $\star$ We are "forced" to move forwards to $\mathcal{P}^N(X;Y)$
- $\star$ $\|\cdot\|_\infty \leftrightarrow \|\cdot\|_v$ on $\mathcal{P}^N(X;Y)$ goes in **totally different** direction

$\star$ A convenient way on how to compute the v-norm of a polynomial

Weighted norms on $\mathcal{P}^N(X; Y)$

⋆ A convenient way on how to compute the v-norm of a polynomial

---

[2022, D., R. Medina]

For every $N \in \mathbb{N}$, there is $s(N) \in (0,1)$ such that for every pair X, Y of Banach spaces and every polynomial $P \in \mathcal{P}^N(X; Y)$, we have

$$\|P\|_v = \sup_{s \in [0, s(N)]} (1 - s^2)\|P\|_s.$$

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[2022, D., R. Medina]

Let X, Y be Banach spaces. Let $N \in \mathbb{N}$ be fixed. Then, for every $s \in (0, 1)$ and every $P \in \mathcal{P}^N(X; Y)$, we have

$$\|P\|_s \geq \left(1 - \sum_{n=1}^{N} (1 - s^n) \cdot \frac{n^n}{n!} \right) \cdot \|P\|_\infty.$$

$\star$ Why do we need such results?

- $\star$ Why do we need such results?
- $\star$ To prove that $\| \cdot \|_\infty$ and $\| \cdot \|_v$ are equivalent norms on $\mathcal{P}^N(X; Y)$

[2022, D. and R. Medina]

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Weighted norms on $\mathcal{P}^N(X; Y)$

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In particular,

$$\| \cdot \|_{s(\alpha,N)}, \qquad \| \cdot \|_\infty \qquad \text{and} \qquad \| \cdot \|_v$$

are all equivalent on $\mathcal{P}^N(X;Y)$.

Weighted norms on $\mathcal{P}^N(X;Y)$

[2022, D. and R. Medina]

$\|\cdot\|_v$ and $\|\cdot\|_\infty$ are equivalent norms on $\mathcal{P}^N(X;Y)$

Remark

$\|\cdot\|_\infty$ and $\|\cdot\|_v$ **cannot** be equivalent on $\mathcal{P}(X;\mathbb{C}) = \bigcup_{N=0}^\infty \mathcal{P}^N(X;\mathbb{C})$.

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Weighted norms on $\mathcal{P}^N(X; Y)$

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> * $\|f_N\|_v = \delta_N \longrightarrow 0$ when $N \to \infty$

There are non-homogeneous polynomials
which attain their *v*-norms but not
the sup-norm and vice-versa

NA_v and NA_∞ are different on $\mathcal{P}^N(X; Y)$

[2022, D. and R.Medina]

Let $1 \leq p < \infty$. For every $k \in \mathbb{N}$ with $k \geq p$,

NA$_v$ and NA$_\infty$ are different on $\mathcal{P}^N(X;Y)$

[2022, D. and R.Medina]

Let $1 \leq p < \infty$. For every $k \in \mathbb{N}$ with $k \geq p$,

* ⋆ there exists $P \in \mathcal{P}^{k+1}(\ell_p; \mathbb{C})$ such that
  * ⋆ $P \in \mathrm{NA}_v \, \mathcal{P}^{k+1}(\ell_p; \mathbb{C})$
  * ⋆ $P \notin \mathrm{NA} \, \mathcal{P}^{k+1}(\ell_p; \mathbb{C})$

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⋆ How do we do that?

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- $\star$ How do we do that?
- $\star$ By proving that there are polynomials which attain their weighted norms for **some** radius but **not** all

Can we talk about denseness?

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$$\star = v \quad \text{or} \quad \star = \infty \Rightarrow \begin{cases} \overline{\mathrm{NA}_v \, \mathcal{P}^N(X;Y)}^{\|\cdot\|_\star} \overset{(?)}{=} \mathcal{P}^N(X;Y) \\ \overline{\mathrm{NA}_v \, \mathcal{A}_u(B_X;Y)}^{\|\cdot\|_\star} \overset{(?)}{=} \mathcal{A}_u(B_X;Y) \end{cases}$$

Denseness: a negative result

Let $N \geq 2$ and $w \in \ell_N$ be an admissible sequence. Then,

$$\overline{\mathrm{NA}\,\mathcal{P}^N(d_*(w,1);\mathbb{C})}^{\|\cdot\|_\infty} \neq \mathcal{P}^N(d_*(w,1);\mathbb{C}).$$

Denseness: a negative result

[2015, Carando and Mazzitelli]

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[2022, D. and R. Medina]

Let $N \geq 2$ and $w \in \ell_N$ be an admissible sequence. Let $X = d_*(w,1)$ and Y be a strictly convex Banach space. There exists an N-homogeneous polynomial $Q \in \mathcal{P}(^N X; Y)$ such that

$$Q \notin \overline{\mathrm{NA}_v\,\mathcal{A}_u(B_X;Y)}^{\|\cdot\|_\infty}$$

In particular,

$$\overline{\mathrm{NA}_v\,\mathcal{A}_u(B_X;Y)}^{\|\cdot\|_\infty} \neq \mathcal{A}_u(B_X;Y)$$

Denseness: a positive result

[2022, D. and R. Medina]

Let X be a uniformly convex Banach space and Y an arbitrary Banach space. Then,

$$\overline{\mathrm{NA}_v \, \mathcal{A}_u(B_X; Y)}^{\|\cdot\|_\infty} = \mathcal{A}_u(B_X; Y)$$

New directions of research

New directions of research

$\star$ Suppose that X satisfies the Radon Nikodým property and Y an arbitrary Banach space. Then,

$$\overline{\mathrm{NA}_v \, \mathcal{A}_u(B_X; Y)}^{\|\cdot\|_\infty} \overset{(?)}{=} \mathcal{A}_u(B_X; Y)$$

[2023, D., R. Medina]

$\star$ A natural question is to know "how large" is the set

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$\star$ $\mathcal{L}(^N c_0, Y) \setminus \overline{\mathrm{NA}(c_0, Y)}^{\|\cdot\|_\infty}$ is $\mathfrak{c}$-spaceable.

[2023, D., Falcó, Jung, Rodríguez-Vidanes]

들어주셔서 감사합니다.
또 만나 뵙기를 바랍니다.

사랑합니다!