DEN Homework # 11

Solve the problems and then bring your work to the lab in the twelfth week of school.

1. Consider the following system.

$$\begin{aligned} 3x - 2y + z &= 7 \\ x - y &= 2 \\ -4x + 2y &= -6. \end{aligned}$$

a) Solve this system using elimination in the way it is used in numerical mathematics, that is, first GEM and then the back substitution.
While doing the elimination, note down the steps that you perform. The form of those notes is up to you, but they should allow you to recreate your elimination.

b) Determine the determinant of the matrix of the system, use the row-echelon form for it.

c) We are given a new system

$$\begin{aligned} 3x - 2y + z &= 4 \\ x - y &= 1 \\ -4x + 2y &= -4 \end{aligned}$$

that has the same left-hand sides as the previous one (what a remarkable coincidence). Apply the row operations that you noted down in part a) just to the vector of the right-hand sides of the new systen. Connect the resulting vector with the reduced upper-triangular matrix from part a) and use the backward elimination to find the solution.
Check by substituting into the given system that you indeed found the right solution.

2. Consider the matrix $A = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}$.
Find its eigenvalues and the spectral radius. Calculate its row norm and column norm.