

A sample of the final exam

1. a) [15 pts] Find the solution of the initial value problem

$$y'' - 5y' + 4y = 34 \sin(x) - 3e^x, \quad y(0) = 6, \quad y'(0) = 5.$$

- b) [5 pts] A supplementary question, see the web. For instance, find the its stationary solutions and study their stability.

2. a) [10 pts] Find a general solution of the equation

$$3y' = -\frac{2}{x^3y^2}$$

and the solution satisfying $y(1) = -1$.

- b1) [10 pts] Find a general solution of the system

$$\begin{cases} y_1' = -6y_1 + y_2, \\ y_2' = -8y_1 \end{cases}$$

using the matrix approach.

- Write the solution in the form $y_1(x) = \dots$, $y_2(x) = \dots$.
- Stability of the stationary solution $(y_1, y_2) = (0, 0)$, justify your answer.

- b2) [10 pts] Use the variation method to find a general solution of the equation

$$y' - 3y = 2e^x.$$

3. a) [5 pts] For the equation

$$y' = x(y - \sin(x))$$

sketch its slope field and determine its stationary solutions (if any exist).

- b) [6 pts] Judge suitability of basic three solution methods for the equation

$$y' + \frac{1}{x}y = \frac{1}{x}.$$

- c) [9 pts] A somewhat harder question, see the web.

4. a) [10 pts] Consider the integral

$$\int_{-3}^3 \frac{10}{x^2 + 1} dx.$$

- Use the trapezoid method with partition size $n = 3$ to estimate its value.
- Explain the principle of this method with a picture (could be symbolic, not necessarily with the given function).
- Indicate in it the error of the method.

- b) [10 pts] Introduce the algorithm for Euler method for solving differential equations

$$y' = f(x, y).$$

with initial condition $y(a) = y_0$ on an interval $[a, b]$. Explain its principle with a picture. State the order of this method and explain what it means.