Week 2 Group Video Lab

The Cup-Filling Continuity Lab

A challenge about limits and continuity.

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The group of five

Form a group of exactly five students. Each student has one role and must speak:

1. **Experiment coordinator:** presents the setup and the filling process.
2. **Data recorder:** presents the measurements and the graph.
3. **Limit analyst:** explains the two one-sided limits and continuity.
4. **Investigator:** predicts and verifies the target-height crossing.
5. **Conclusion speaker:** explains what the experiment shows and what the measurements cannot prove.

The experiment and the data

Use a transparent container, lightly coloured water, a ruler, tape and a phone. Keep the phone fixed and away from the water.

1. Attach the ruler vertically and mark a target height H with tape.
2. Pour water continuously for about 15 seconds while recording.
3. Define $h(t)$ as the water height, in centimetres, at time t.
4. Record at least seven time–height pairs and draw an approximate graph.
5. Choose an interior time a and include the measurements at

$$a-1, \quad a-0.5, \quad a, \quad a+0.5, \quad a+1.$$

Show the table and graph in the video. No separate written report is required.

One-sided limits at the chosen time

Use the frames immediately before and after a to estimate

$$\lim_{t \to a^-} h(t) \qquad \text{and} \qquad \lim_{t \to a^+} h(t).$$

1. Show the values $h(a-1)$, $h(a-0.5)$, $h(a+0.5)$ and $h(a+1)$.
2. State your estimates of the left-hand and right-hand limits.
3. Compare them with the measured value $h(a)$.
4. Conclude whether your experiment supports

$$\lim_{t \to a^-} h(t) = \lim_{t \to a^+} h(t) = h(a).$$

The target-height challenge

Choose times $t_1 < t_2$ for which the tape height satisfies

$$h(t_1) < H < h(t_2).$$

1. Explain why continuity and the Intermediate Value Theorem guarantee at least one time $c \in (t_1, t_2)$ such that $h(c) = H$.
2. Predict the value of c from your table or graph.
3. Find the frame at which the water reaches the tape mark.
4. Compare the observed time with your prediction.

The three-minute video

0:00–0:30	Show the setup and a short part of the experiment.
0:30–1:00	Present the measurements and the graph of h.
1:00–1:40	Explain the one-sided limits and continuity at a.
1:40–2:30	Apply the Intermediate Value Theorem and verify the crossing.
2:30–3:00	State the group's conclusion and the limitations of the data.

All five students must speak. Short cuts are allowed, but special editing is not required.

Submission checklist

Submit one video file or one accessible video link for the whole group. Check that it includes:

- a visible experiment and at least seven time–height measurements
- a labelled graph of h
- both one-sided limits, $h(a)$ and the continuity conclusion
- the target-height application of the Intermediate Value Theorem
- a short conclusion about what the measurements show
- the names of all five students.

The video should be sent to: SHELDON.DANTAS@GMAIL.COM