

ELEMENTARY FUNCTIONS

Exercise 1. Find the domain of each of the following functions:

a) $f(x) = \sqrt{\frac{x-2}{x+2}}$

c) $f(x) = \sqrt{\frac{x}{1-|x|}}$

d) $f(x) = \log(|x| - 7)$

b) $f(x) = \log\left(\frac{x^2-5x+6}{x^2+4x+6}\right)$

e) $f(x) = \frac{[\log_3(x) + \log_{10}(x+3)](x+1)}{\sqrt{x-3}}$

Exercise 2. Determine whether each of the following functions is even or odd:

a) $f(x) = e^x + e^{-x}$

d) $f(x) = \sin(|x|)$

b) $f(x) = e^x - e^{-x}$

e) $f(x) = \cos(x^3)$

c) $f(x) = |x+1| - |x-1|$

f) $f(x) = \log_5(3x^2 + 1)$

Exercise 3. Determine the monotonicity¹ of the following functions:

a) $f(x) = \arctan(\log(x)), \forall x > 0,$

c) $f(x) = \frac{1}{x+e^x}, \forall x > 0,$

b) $f(x) = x + \sqrt{x}, \forall x > 0,$

d) $f(x) = \sin(\cos(x)), \forall x \in [0, \pi/2].$

Exercise 4. Verify that the identity $a^{\log(b)} = b^{\log(a)}$ holds for every pair of positive numbers a and b .

Exercise 5. Prove that $\log\left(x + \sqrt{1+x^2}\right) + \log\left(\sqrt{1+x^2} - x\right) = 0$.

Exercise 6. Solve the equation $x^{\sqrt{x}} = (\sqrt{x})^x$.

Exercise 7. Simplify the following expressions:

a) $a^{\log(\log a) / \log a},$

b) $\log_a(\log_a(a^{a^x})).$

Exercise 8. Use the sine and cosine addition formulas to calculate the following values:

a) $\sin(7\pi/12)$ and $\cos(\pi/12),$

b) $\tan(11\pi/6).$

Hint: Recall that

$$\frac{\pi}{3} + \frac{\pi}{4} = \frac{4\pi}{12} + \frac{3\pi}{12} = \frac{7\pi}{12}, \quad \frac{\pi}{3} - \frac{\pi}{4} = \frac{4\pi}{12} - \frac{3\pi}{12} = \frac{\pi}{12}, \quad \frac{11\pi}{6} = 2\pi - \frac{\pi}{6}.$$

Exercise 9. Determine whether the following identities are true:

a) $\arccos(\cos(\pi/4)) = \pi/4,$

c) $\arctan(\tan(3\pi/2)) = 3\pi/2,$

b) $\arcsin(\sin(10)) = 10,$

d) $\sqrt{x^2} = x.$

¹Look up on the internet what is the meaning of this and solve the exercises.