

Week 4 Group Video Lab

The L'Hospital Relay

One limit, one graph and five active participants.

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The mission

Work in a group of exactly five students. Create one short investigation of a limit that requires **exactly two** applications of L'Hospital's rule.

Your group will produce:

- ★ one calculation, with every hypothesis checked;
- ★ one interactive Desmos or GeoGebra (or any program you like) graph;
- ★ one video of at most three minutes.

Choose your group's limit

Choose one of the following families of functions:

$$R(x) = \frac{e^{ax} - 1 - ax}{bx^2}, \quad R(x) = \frac{\ln(1 + ax) - ax}{bx^2}, \quad R(x) = \frac{1 - \cos(ax)}{bx^2}.$$

Use positive integers a and b connected to your group. For example:

- a can be the number of countries represented in the group;
- b can be the number of languages spoken by the group.

Your limit is taken as $x \rightarrow 0$. Before differentiating, explain why the original quotient has the form $0/0$.

Build only one interactive graph

In one Desmos or GeoGebra (or in any app you like) coordinate plane, enter

$$R_0(x) = \frac{N(x)}{D(x)}, \quad R_1(x) = \frac{N'(x)}{D'(x)}, \quad R_2(x) = \frac{N''(x)}{D''(x)}.$$

Use the visibility buttons to show the three expressions **one at a time**. Your single graph must include:

- a suitable window near $x = 0$;
- the three clearly labelled expressions;
- the horizontal line $y = L$, where L is your final limit;
- an open point at $(0, L)$ to represent the limiting value.

Do not create separate graphs or separate scenes.

Complete the mathematical relay

Your explanation must contain the following four stages:

1. Verify that $R_0(x)$ has the form $0/0$.
2. Apply L'Hospital's rule once and check that $R_1(x)$ is still $0/0$.
3. Apply the rule a second time and calculate the limit of $R_2(x)$.
4. Explain why applying L'Hospital's rule a third time would be invalid.

The graph should support the calculation by showing that R_0 , R_1 and R_2 approach the same height.

The challenge is not only knowing when to start, but also when to stop.

Five students, five essential jobs

Every student must speak and must be responsible for one visible part of the work.

- Student 1** Introduces the group parameters and verifies the first $0/0$ form.
- Student 2** Performs and explains the first application of L'Hospital's rule.
- Student 3** Performs the second application and calculates the final limit.
- Student 4** Builds and operates the single interactive graph during the video.
- Student 5** Explains why the process must stop and checks the complete argument.

The three-minute video

- 0:00–0:25 Introduce the parameters, quotient and limiting point.
- 0:25–0:55 Verify $0/0$ and perform the first application.
- 0:55–1:25 Check the new form and perform the second application.
- 1:25–2:15 Operate the graph and compare R_0 , R_1 and R_2 .
- 2:15–2:50 Explain why a third application would be invalid.
- 2:50–3:00 State the final limit clearly.

All five students must speak. Keep the graph and calculations legible on screen.

What to submit

Submit one item for the whole group containing:

- ★ one accessible Desmos or GeoGebra graph link;
- ★ one video file or accessible video link, at most three minutes;
- ★ the names of all five students in both the graph notes and the video;
- ★ the values of a and b and their connection to the group;
- ★ the original quotient and both derivative quotients.

Everything needs to be sent to: sheldon.dantas@gmail.com