Homework #7

The Taylor Signal Emulator

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Calculus 1 · Winter term 2026–2027

The mission

Work in a group of exactly five students. Your group will replace a damped signal near $t = 0$ by a Taylor polynomial.

You will produce:

- one fourth-order Taylor polynomial obtained from explicit derivatives;
- one rigorous Lagrange error estimate;
- one interactive Desmos or GeoGebra graph;
- one live presentation of at most three minutes.

One member of each group will be selected at random to present the complete work next Wednesday.

The signal and the mathematical task

Consider the normalized damped signal

$$s(t) = e^{-t}\cos t, \qquad t \geq 0.$$

Your group must:

1. calculate $s'(t), s''(t), s^{(3)}(t), s^{(4)}(t)$ and $s^{(5)}(t)$ one by one;
2. construct the fourth-order Maclaurin polynomial $P_4(t)$;
3. use $P_4(0.5)$ to approximate $s(0.5)$;
4. find a Lagrange upper bound for $|s(0.5) - P_4(0.5)|$;
5. calculate the actual error and compare it with the bound.

The error certificate

Your error argument must identify all of the following:

- the interval between the centre 0 and the point 0.5;
- the derivative that appears in the Lagrange remainder for P_4;
- a justified constant M satisfying $\left|s^{(5)}(t)\right| \leq M$ on that interval;
- the resulting numerical upper bound.

Do not use a ready-made Taylor-series formula or a computer algebra command to differentiate the function. You may use a calculator only after obtaining the polynomial approximation.

One interactive graph and three views

Create one Desmos or GeoGebra graph containing

$$s(t), \qquad P_0(t), P_1(t), \ldots, P_4(t),$$

together with

$$E(t) = |s(t) - P_4(t)|$$

and the Lagrange bound as a function of t on $[0, 0.5]$.
During the live presentation, use the same graph in three ways:

1. Begin close to $t = 0$ with the labels hidden. Ask which curve is the signal.
2. Reveal the labels and zoom out until P_4 visibly separates from s.
3. Show the error curve below the Lagrange bound on $[0, 0.5]$.

Use visibility buttons. Do not prepare three separate graphs.

The random presentation rule

The presentations will take place at the end of class on **Wednesday, November 11**.

- Each group must be ready with its complete presentation.
- 5 students will selected at random, each one from different groups.
- The selected students will come to the blackboard and present for at most 3 min.
- Come ready to share a Desmos or GeoGebra graph.
- The ones who want to get "homework accepted", previously to Wednesday 11th, need to send an email saying the group is ready to participate. The email is: sheldon.dantas@gmail.com. Therefore, either everybody gets ✓ on Wednesday 11th or everybody loses it.