Calculus 1

Group Activity: Taylor Polynomial and the Lagrange Remainder

Instructions. Work in groups of five. You have 45 minutes to solve and discuss both problems. Every member must understand every line of the solutions because the presenting groups will be selected only after the working time has ended.

Problem 1

Let

$$f(x) = e^x \cos x.$$

(a) Calculate the derivatives needed to construct the fourth-order Taylor polynomial P_4 of f centred at $a = 0$.

(b) Use the Lagrange remainder to prove that

$$|f(x) - P_4(x)| \leqslant \frac{\sqrt{2e}}{960} \qquad \text{for every } x \in \left[-\frac{1}{2}, \frac{1}{2}\right].$$

Problem 2

We want to decide which of the two numbers

$$e^\pi \qquad \text{and} \qquad \pi^e$$

is larger. Define

$$F(x) = \frac{x}{e} - \ln x, \qquad x > 0.$$

(a) Calculate the derivatives needed to construct the fourth-order Taylor polynomial $P_4(x)$ of F centred at $a = e$. Write the polynomial explicitly as a function of x before substituting $x = \pi$.

(b) Evaluate $P_4(\pi)$ and use it to approximate

$$G = \ln\left(\frac{e^\pi}{\pi^e}\right) = \pi - e \ln \pi.$$

(c) Use the Lagrange remainder to prove that the error in the approximation of G is smaller than 5×10^{-5}.

(d) Use the resulting certified approximation to decide which of e^π and π^e is larger.

Hint. Observe that

$$G = eF(\pi).$$